REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. REPORT DATE *(DD-MM-YYYY)*	**2. REPORT TYPE**	**3. DATES COVERED** *(From - To)*
15-02-2012	final report	04/15/2009--11/30/2011

4. TITLE AND SUBTITLE
Optimization Algorithms and Equilibrium Analysis for Dynamic Resource Allocation

5a. CONTRACT NUMBER

5b. GRANT NUMBER
FA9550-09-1-0319

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)
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5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Dept. of Electrical and Computer Engineering
University of Minnesota
Minneapolis, MN 55455

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)
AFOSR
875 N. RANDOLPH ST.
SUITE 325
ARLINGTON, VA 22203

10. SPONSOR/MONITOR'S ACRONYM(S)

11. SPONSOR/MONITOR'S REPORT NUMBER(S)
AFRL-OSR-VA-TR-2012-0536

12. DISTRIBUTION / AVAILABILITY STATEMENT
DISTRIBUTION A: APPROVED FOR PUBLIC RELEASE

13. SUPPLEMENTARY NOTES

14. ABSTRACT
This research project is focused on developing computational game theoretic techniques to tackle the design and computation of equilibrium power allocation/control schemes for a complex heterogeneous communication system. We have explored the use of noncooperative game theory, MPEC (Mathematical Programs with Equilibrium Constraints) to formulate and study the spectrum management problem. We have also addressed the design and computation complexities of equilibrium power spectrum policies in various game theoretic frameworks and proposed efficient distributed algorithms for dynamic power spectrum design.

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			**17. LIMITATION OF ABSTRACT**	**18. NUMBER OF PAGES**	**19a. NAME OF RESPONSIBLE PERSON**
a. REPORT	**b. ABSTRACT**	**c. THIS PAGE**		12	**19b. TELEPHONE NUMBER** *(include area code)*

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std. Z39.18

AFOSR Final Report
February 2012

Optimization Algorithms and Equilibrium Analysis for Dynamic Resource Allocation

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Contents

1 List of illustrations

2 Statement of the problem studied

This research project is focused on developing computational game theoretic techniques to tackle the design and computation of equilibrium power allocation/control schemes for a complex heterogeneous communication system. We have explored the use of noncooperative game theory, MPEC (Mathematical Programs with Equilibrium Constraints) to formulate and study the spectrum management problem. We have also addressed the design and computation complexities of equilibrium power spectrum policies in various game theoretic frameworks and proposed efficient distributed algorithms for dynamic power spectrum design.

The following theoretic aspects of the DSM (Dynamic Spectrum Management) problem have been studied.

- We have considered a Nash game formulation to analyze the impact of a hostile jammer to the design and computation of equilibrium power allocation strategies and to the overall system performance. Specifically, we analyzed the convergence behavior of the best-response strategy under a weak diagonal dominance condition. We will use Lyapunov convexity theorem to analyze the (asymptotic) min-max duality of DSM problem and use it to develop polynomial time approximation schemes to compute an ϵ-approximate equilibrium solution.
- We have considered a Stackelberg game formulation of the DSM problem whereby a central coordinator picks a set of upper bounds for the maximum allowed power levels in each tone (i.e., a spectral mask) while the users play a Nash game to reach an equilibrium subject to individual power budget constraint and the joint spectral mask constraint (specified by the central coordinator). We studied the existence of an equilibrium solution and propose computational techniques to determine the optimal spectrum sharing policies under this MPEC framework. Moreover, we determined the complexity status of this Stackelberg game and identify sub-classes which are polynomial time solvable. We also developed efficient distributed approximation algorithms for power control by combining FDMA and IWFA in different frequency bands depending on the level of interference.

- We have also considered the problem of weighted sum rate optimization in a MIMO interfering multiple access channel (IMAC). We proposed to jointly optimize the users transmitter structures as well as their base station (BS) associations. This approach enables the users to avoid congested BSs and can improve system performance as well as user fairness. We formulate the problem into a noncooperative game, and develop an algorithm that allows the players to distributedly reach the Nash Equilibrium (NE) of the game. We showed that every NE of the game is a stationary solution of the weighted sum rate optimization problem, and proposed an algorithm to compute the NE of the game. Simulation results showed that the proposed algorithm performs well in the presence of BS congestion.

These results provide not only useful insights on the structure of optimal spectrum sharing strategies but also simple distributed algorithms for power control and spectrum sensing that are practically implementable in a large scale military communication system.

3 Summary of the most important results

Significant progress has been made on several fronts:

3.1 A Stackelberg Game Approach to Distributed Spectrum Management

We consider a cognitive radio system with one primary (licensed) user and multiple secondary (unlicensed) users. Given the interference temperature constraint, the secondary users compete for the available spectrum to fulfill their own communication need. Borrowing the concept of price from market theory, we develop a decentralized Stackelberg game formulation for power allocation. In this scheme, the primary user (leader) announces prices for the available tones such that a system utility is maximized. Using the announced prices, secondary users (followers) compete for the available bandwidth to maximize their own utilities. We show that this Stackelberg game is polynomial time solvable under certain channel conditions. When the individual power constraints of secondary users are inactive (due to strict interference temperature constraint), the proposed distributed power control method is decomposable across the tones and unlike normal water-filling it respects the interference temperature constraints of the primary user. When individual power constraints are active, we propose a distributed approach that solves the problem under an aggregate interference temperature constraint. Moreover, we propose a dual decomposition based power control method and show that it solves the Stackelberg game asymptotically when the number of tones becomes large.

We have examined the performance of the proposed algorithms in terms of achieved sum rate and power consumption. We consider a multiuser system with 1 primary user, 8 secondary users sharing 8 frequency tones. Equal weights and power budgets are considered across the users. The channel coefficients are drawn from Rayleigh random variable and in order to satisfy the weak interference condition, the cross-talk coefficients are scaled by a factor of 10%. The noise power is set to one at all receivers. In the first numerical experiment, we compare the proposed algorithms in terms of achieved sum rate while the power budget of the secondary users varies. We fix the interference temperature constraint to -5 dB and plot the total sum rate versus the power budget constraint of the secondary users (see Fig. 1). Note that since we normalized the channel coefficients from the secondary users to the primary user, we should scale the interference temperature constraint accordingly. For example, high interference temperature constraint corresponds to the case where the primary and the secondary users are far away.

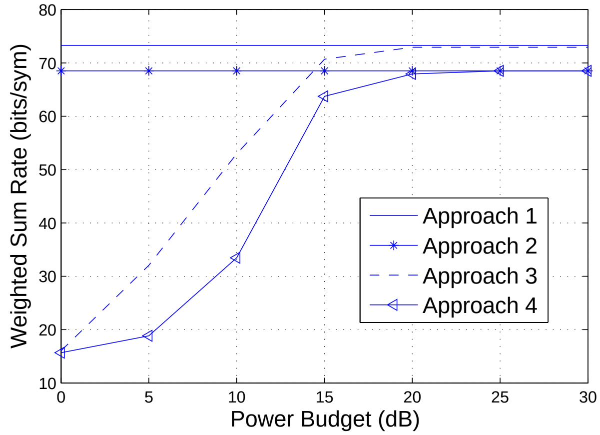

Figure 1. Sum rate achieved using different methods, interference temperature level $= -5$ dB

In the second numerical experiment, we consider equal power budget of 15 dB for all users and plot the sum rate achieved by the proposed methods for different values of interference temperature constraint (see Fig. 2). Although approach 1 leads to a better sum rate, it consumes more power as compared to the other approaches (see Fig. 3 for comparison in terms of the total power consumption, i.e., $\sum_{k=1}^{K}\sum_{n=1}^{N} s_k^n$). Furthermore, it fails to respect the power budget constraint of individual users.

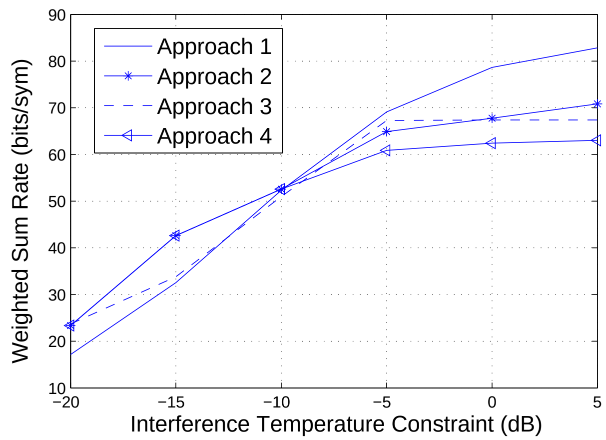

Figure 2. Sum rate achieved using different methods, user power budget fixed at 15 dB

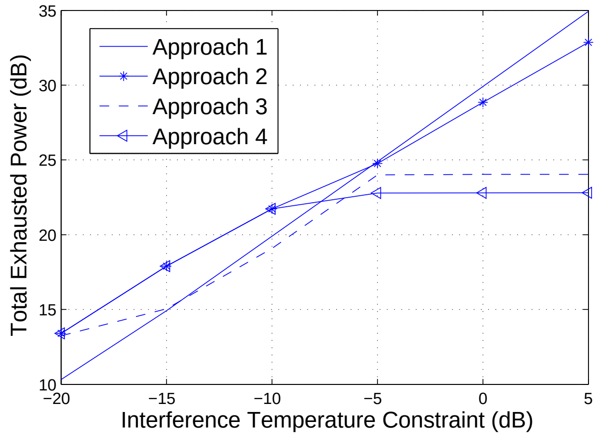

Figure 3. Total power consumption ($\sum_{n=1}^{N}\sum_{k=1}^{K} s_k^n$), used power budget fixed at -5 dB

3.2 Joint Transmit Covariance Optimization and Base Station Selection in Uplink Mimo Network: A Game Theoretic Approach

We consider a general MIMO-IMAC in which a set of transmitters send data to their intended BSs at the same time. Both the transmitters and the BSs are equipped with multiple antennas. Such channel model is important as it can accurately describe many practical networks, e.g., the uplink of multicell heterogenous networks (HetNet), in which the transmitters may represent either mobile users or the relays. The MIMO-IMAC is a generalization of the MIMO interference channel (IC), in which each BS has only one associated transmitter. We are interested in the optimal design of both the users' linear transmit beamformer and the overall user-BS assignment to achieve spectral efficiency and load balancing for the entire network.

The problem considered in this work is important for the design of the envisioned heterogenous networks. In such networks, the cell sizes will become smaller, and the deployment of access points such as macro/pico/femto BSs will become denser. The users are often simultaneously covered by multiple BSs with different capabilities and load status. The traditional strongest BS association approach is insufficient for congestion management and fairness provisioning in such heterogenous networks. Consequently, optimal BS assignment becomes a crucial aspect in the overall system performance optimization.

The problem of optimal transceiver design of MIMO-IC has been extensively studied recently. Many authors formulate the transmission covariance matrix optimization problem into a noncooperative game, in which the transmitters/users compete with each other for transmission. Some authors take into consideration the robustness issue of the problem. Each user aims at selfishly maximize their own transmission rate, while treating all other users' interference as noise. Simple distributed algorithms with convergence guarantees are derived, but the outcome of the game is inefficient in terms of system performance. This is due to both the lack of coordination among the transmitters/users and the inability to take into consideration some more general utility functions that balances the user fairness and system throughput.

Instead of the competitive design, one can optimize the system performance measured by some suitable system utility functions. Unfortunately, many of the problems in this category have been proven to be NP-complete in various settings. As a result, many authors focus on developing high quality algorithms to compute sub-optimal solutions for these problems, e.g., in MIMO-IC, MISO-IC and MIMO-IC with single stream per user. A local linear approximation algorithm was proposed in the literature based on the first order Taylor expansion of the weighted sum rate objective. This algorithm allows the transmitters/users to update their transmission covariance matrices by solving a series of convex optimization problems. However rigorous convergence results for this algorithm are not established, that is, the weighted sum rate (or the the users' transmit covariance) may not converge to a local optimal solution. It it worth noting that all the above cited algorithms are only designed for (weighted) sum rate optimization, and it is not clear how they may be generalized to deal with other utility functions that takes user fairness into consideration. There is only a few recent works focusing on the problem of optimal design for *general* utility maximization. Recently we proposed a cyclic ascend method to optimize the linear transmit precoder in a MISO network to optimize a system-wide smooth utility function. Moreover, we proposed a weighted Minimum Mean Square Error (WMMSE) algorithm in which the transmitters and receivers iteratively update their linear transmission and receiving strategies to optimize the system utility function. The authors show that as long as the system utility function satisfies some regularity conditions, their algorithm is guaranteed to converge to a stationary point of the problem.

All the above cited works aim at optimizing the linear transceiver structures *assuming that the transmitter-receiver association is known and fixed*. In our considered IMAC setting though, BS assignment becomes an important optimization variable for achieving spectral efficiency and load balancing for the entire network. The problem of joint cell site selection and power allocation in the traditional CDMA based network has been first considered in the mid 1990's and later in a game theoretical perspective. In these works, the users optimize their uplink power levels (which is a scalar variable) as well as their BS association. The phenomenon of "cell breathing" is observed in which the size of the cells dynamically changes according to the congestion levels. Recent works in this area have considered the joint BS selection and vector power allocation in OFDMA networks in which the BSs operate on non-overlapping spectrum. The users compete with each other for resources in each cell, in the meantime they can also freely choose to use any (non-interfering) BS in the network. However, it is not clear how these works can be generalized to the considered MIMO-IMAC scenario.

In this work, we consider the problem of *joint* optimization of linear transmitter and BS assignment in a MIMO-IMAC. This network setting is more general than those considered in the previously mentioned works, consequently their approaches cannot be applied here. We start by investigating the complexity of finding the global optimal joint linear precoder and the user-BS assignment that maximize the system weighted sum rate. Unfortunately, we show that this task is computationally intractable in general. This negative result motivates us to find computationally efficient algorithms that are capable of identifying approximately optimal solutions to the joint problem with general utility functions. To this end, we formulate the joint linear precoder design and user-BS assignment problem in a game theoretical framework. We investigate the properties of the Nash Equilibrium (NE) of this game, and propose efficient and distributed algorithms to reach these NEs.

The main contribution of this work is summarized as follows.

- We present the complexity analysis for the joint problem when the weighted sum rate utility is used. This result is intrinsically different from the existing complexity results for resource management in wireless communications. In those cases, the hardness of the problem is mainly due to the possibility of strong interference among the users and the ensuing difficulty in identifying the set of users/antennas that needs to be shut down. In contrast, in our problem the hardness lies in its mixed (discrete and continuous) formulation.

- We propose a novel game theoretical formulation to find an approximately optimal solution for the joint problem. In the proposed game both the transmitters and the BSs are the players. Each transmitter aims at finding the best linear precoder as well as the least congested BS for transmission. Each BS computes a set of optimal prices to charge the transmitters for causing interference. These prices serve to coordinate the behavior of the transmitters so that they do not cause excessive interference. Our formulation and algorithms lead to desirable operation points of the network such that: 1) the system is stable, in the sense that no single transmitter is willing to change its BS association; 2) the transmission strategies of the users are efficient, in the sense that they achieve a local optimal solution of the overall system level utility maximization problem.

- When the user-BS assignment is *fixed*, the proposed game reduces to a simpler precoder optimization game. The users are again charged with a set of prices that reflect their (negative) influence to other users. Our pricing formulation is a generalization of previous works for interference pricing to MIMO-IMAC setting with general system utility functions.

We have numerically tested the proposed resource allocation algorithms in extensive computer simulations. Consider a network with 7 BSs and 16 users. The distance between adjacent BSs is 200 meters (representing a dense network of small cell size). Let $d_{q,n}$ be the distance between BS q and user n. The entries of the channel $\mathbf{H}_{q,n}$ are generated from distribution $\mathcal{CN}(0, \sigma^2_{q,n})$. The standard deviation is calculated by $\sigma_{q,n} = (200/d_{q,n})^{3.5} L_{q,n}$, where $10 \log 10(L_{q,n}) \sim \mathcal{N}(0, 8)$ models the shadowing effect. We fix the environment noise power as $\sigma_q = 1$ for all $q \in \mathcal{Q}$, and define the SNR as $10 \log_{10} \bar{p}_n$.

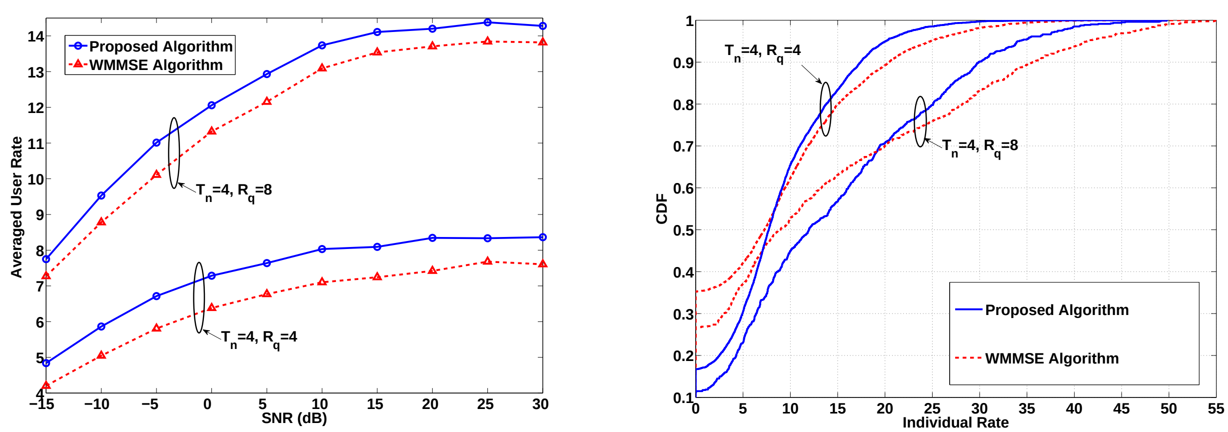

Figure 4. Comparison of the performance of the proposed algorithm and the WMMSE algorithm when the sum rate utility is used. $N = 16$, $Q = 7$, users are all located at the cell edges. Left: comparison of the users' averaged rates. Right: comparison of the CDF of users' achieved rates.

We first consider a scenario in which the users are all located at the cell edges, and one of the BSs is congested. We place half of the users uniformly at the cell edges of BS 1, which is within $d_{1,n} \in [90,\ 100]$ meters. We place the rest of the users randomly at the cell edges of other BSs. For

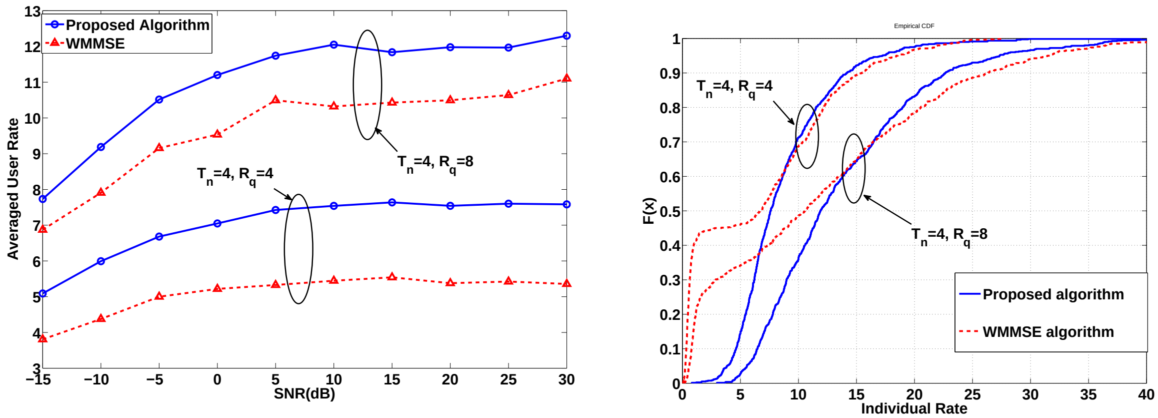

Figure 5. Comparison of the performance of the proposed algorithm and the WMMSE algorithm when the proportional fair utility is used. $N = 16$, $Q = 7$, users are all located at the cell edges. Left: comparison of the users' averaged rates. Right: comparison of the CDF of users' achieved rates.

the WMMSE algorithm, we let the users associate to the BSs with the highest channel magnitude (in terms of the 2-norm of the channel matrices). For our proposed algorithm, we limit the users to be able to choose their association among the three strongest BSs. We initiate our algorithm by assigning the users to their respective strongest BSs. Fig.4 compares the performance of the algorithms when the sum rate utility is used (i.e., $f_n(R_n) = R_n, \ \forall \ n \in \mathcal{N}$). Each point on this figure is averaged over 100 random generation of users' positions and channel coefficients. The left panel of Fig. 4 compares the users' averaged rates achieved by different algorithms. The right panel of Fig. 4 compares the CDF of the individual rates of the two algorithms when $SNR = 30dB$. Fig. 5 shows the same comparison but with the proportional fair (PF) utility as the system utility function, i.e., $f_n(R_n) = \log(R_n), \ \forall \ n \in \mathcal{N}$. Clearly for both cases the proposed algorithm achieves higher spectrum efficiency and fairer rate allocation.

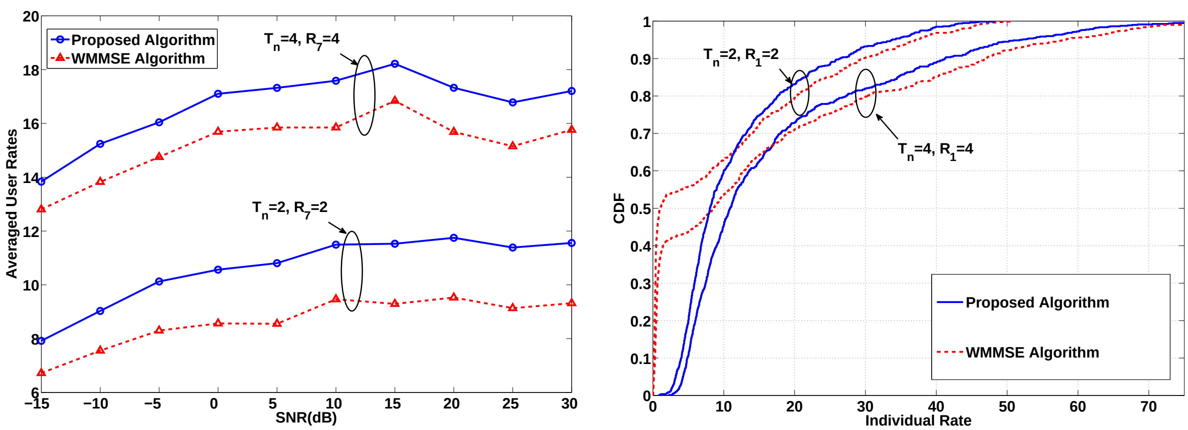

Figure 6. Comparison of the performance of the proposed algorithm and the WMMSE algorithm in a HetNet with different BS capabilities. The proportional fair utility is used. $N = 16$, $Q = 7$. Half of the users are uniformly distributed in cell 1, all other users are uniformly distributed in other cells. Three neighboring BSs have 10 receiving antennas, and all other BSs have the same number of antennas as BS 1. Left: comparison of the users' averaged rates. Right: comparison of the CDF of users' achieved rates.

Comparing Fig. 4 and Fig. 5, we see that the proposed algorithm is more effective when we take into consideration the users' fairness. This observation is intuitive as when the sum rate utility is used, the cell edge users that experience weak channels are largely neglected (i.e., forced to shut down), no matter which BS they connect to. In contrast, when the fairness is incorporated into the problem, every user has to be served. In this case assigning the weak users to less congested BSs effectively reduces the interference level (hence the congestion level) of the congested BS, hence enhancing both fairness to the weaker users and the spectral efficiency of the entire network.

3.3 Power Allocation in Multi-Channel Cognitive Radio Networks With Channel Assembling

Spectrum access in Cognitive Radio Networks (CRNs) can be implemented in an Opportunistic Spectrum Access (OSA) manner, where SUs transmit over a frequency band only if none of the PUs is transmitting in that band. By utilizing spectrum sensing, the SUs can decide to transmit if the sensing result indicates that all PU transmitters are inactive at this band.

In distributed CRNs with OSA approach, Medium Access Control (MAC) protocols usually work in a competing manner whereby the SUs compete for access opportunities, with the winning SU using the available channels while other SUs have to wait for the next competition. When multiple available channels exist, channel assembling technique can be utilized by the winner in order to support higher data rate and further improve spectrum utility. Traditionally, waterfilling is adopted for power allocation among multiple channels. However, this approach may lead to high probability of collision between SU and PU activities. If such collision happens, i.e., PUs appear during an SU packet transmission, SUs must release the channel immediately in order to make room for PUs, resulting a possible cost to SUs. Recently, some researchers introduced a risk-return model for SUs in which the cost of this collision in a given band is modeled as a rate loss depending on the power level allocated to this band. Under this model, the optimal power allocation strategy turns out to be similar to the traditional waterfilling. However, in practice, the full impact of such collision is much more than just the wasted transmission power or the associated rate loss. It includes other important ramifications, such as the resulted SU packet loss, the delay and the overhead in the handshake process between SU communication pairs. Hence, modeling this collision just as a rate loss is insufficient. In this work, we directly minimize or constrain the collision probability. Specifically, we consider two optimal power allocation problems for the case where SUs access the channels in a competing manner and only the winner can utilize the vacant channels for packet transmission after competition. One problem is to minimize the collision probability of an SU packet with PUs. The other one is to maximize the data rate given the upper bound of SU packet collision probability.

Consider power allocation for Secondary User (SU) packet transmissions over multiple channels with variable Primary User (PU) arrival rates in cognitive radio networks. Two problems are studied in this work: The first one is to minimize the collision probability with PUs and the second one is to maximize the data rate while keeping the collision probability bounded. It is shown that the optimal solution for the first problem is to allocate all power onto the best channel based on a certain criterion. The second problem with a per-channel power budget constraint is proven to be NP-hard and therefore a pseudo-polynomial time solution for the problem is proposed. When a total power budget for all channels is imposed in the second problem, a computationally efficient algorithm is introduced. The proposed algorithms are validated by numerical experiments.

4 List of Publications Supported by this Project

(I) **Manuscripts submitted, but not published**

(1) "Joint Linear Precoder Optimization and Base Station Selection for an Uplink MIMO Network: A Game Theoretic Approach"

- *Authors:* Mingyi Hong and Zhi-Quan Luo
- Submitted to *IEEE Journal of Selected Areas of Communication.*

(II) **Papers published or accepted for publication in peer-reviewed journals**

(1) "A Stackelberg Game Approach to Distributed Spectrum Management"

- *Authors:* Razaviyayn, M., Luo, Z.-Q., Tseng, P. and Pang, J.-S.
- *Mathematical Programming*, Series B, Vol. 129, pp. 197-224, 2011.

(2) "Distributed Optimization in an Energy-Constrained Network"

- *Authors:* Razavi, A. and Luo, Z.-Q.
- Accepted for publication in *IEEE Transactions on Information Theory, 2011.*

(III) **Papers published in conference proceedings**

(1) "Joint Linear Precoder Optimization and Base Station Selection for an Uplink MIMO Network: A Game Theoretic Approach"

- *Authors:* Mingyi Hong and Zhi-Quan Luo
- Proceedings of IEEE International Conference on Acoustics, Speech, and Signal Processing, 2012.

(2) "Power Allocation In Multi-Channel Cognitive Radio Networks With Channel Assembling"

- *Authors:* Lei Jiao; Razaviyayn, M.; Enbin Song; Zhi-Quan Luo; Li, F.Y.
- Proceedings of IEEE Signal Processing Advances in Wireless Communication, June 2011.